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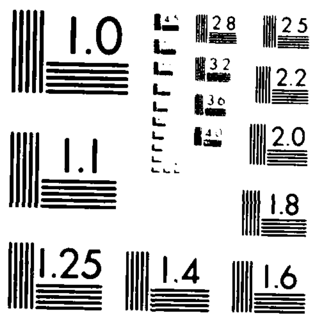

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M.L. Burrows

Channel Tracking Errors and the Adaptive Multiple-Beam Antenna

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FOR THE COMMANDER

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CHANNEL TRACKING ERRORS AND THE ADAPTIVE MULTIPLE-BEAM ANTENNA

M.L. BURROWS

Group 61

TECHNICAL REPORT 659

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LEXINGTON MASSACHUSETTS

Abstract

The ability of an adaptive antenna to reject wide-band interference sources is limited by the differences between the transfer functions of the different channels. An examination of the quantitative relationship between the rejection and the tracking errors shows that the multiple-beam antenna (MBA) achieves greater rejection than an array antenna. This is because, in forming a null in the direction of an interference source, the MBA can usually simply turn off the antenna beam (or beams) principally affected by the source. Further rejection is achieved by cancelling the residual interference signal carried into the antenna on the sidelobes of other beams. Thus it is only at the sidelobe level that the effects of channel tracking errors are felt. The array, on the other hand, must depend upon cancellation to provide its total rejection.

A simple formula exists, and is presented in the text, relating the interference cancellation to the rms mismatch between the channel transfer functions.

CONTENTS

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ILLUSTRATIONS

I. INTRODUCTION

An adaptive antenna is, in practice, unable to attain perfect rejection of an interference source. There are many hardware imperfections which prevent it.[1] One of the more serious of these is the unavoidable difference between the transfer function of one signal channel and that of another. By properly weighting and summing the outputs of the two channels, one can achieve perfect cancellation of any particular single-frequency tone carried on both channels. However, if the signal has a finite bandwidth, rather than being a tone, perfect cancellation cannot be attained unless the channel transfer functions are identical over the band -- that is, unless the channel tracking errors are zero.

Attention has been given, in the case of array antennas, to establishing the quantitative relationship between the interference rejection and the channel tracking errors. A simple deterministic model shows that, in the case of one interference source and two channels, the power rejection factor is proportional to the square of the amplitude and phase errors.[2] For a rejection of -25 dB, the errors must be held to about 0.5 dB in amplitude and 2.8° in phase. A more general, statistical, model shows that the rejection is equal simply to the mean square variation of the uncorrelated channel tracking errors.[3] That is,

$$C = \sigma_t^2 \qquad (1)$$

where C is the expected value of the cancellation or rejection of the interference source and is defined as the ratio of the interference power

level at the output of the adaptive antenna after adaption to the power level before adaption, and σ_t is the rms value of the channel transfer function variations that are uncorrelated from channel to channel. This formula is independent of the number of channels and of the number of interference sources (provided that there are fewer sources than channels).

One further necessary refinement in the definition of C is that it is the limiting ratio of interference powers, after and before adaption, as the intrinsic power of the source rises without limit. This refinement is necessary because the cancellation of an adaptive nulling antenna is usually a function of the power of the interfering source.

The results of numerical simulations show Eq. (1) to be accurate, provided certain assumptions are satisfied. The most straightforward set of assumptions is that the adaptive antenna uses the classical Howells-Applebaum[4] algorithm and that the tracking errors of concern are those occurring in the signal channels between the points bridged by the two correlator connections. This case is sketched in Fig. 1, in which all imperfections are lumped mathematically in the transfer functions $H_n(f)$ $(1 \leq n \leq M)$ and the weighting and summing circuits are assumed to be perfect. Since the $H_n(f)$ are bypassed in forming successive estimates of performance, the adaptive algorithm is unable to compensate even partially for their tracking errors, apart from a simple adjustment of average gain. In this case, Eq. (1) is accurate.

In contrast, mismatch between the channels due to aperture dispersion, for example, which occurs outside the adaptive loops, can be partially or even

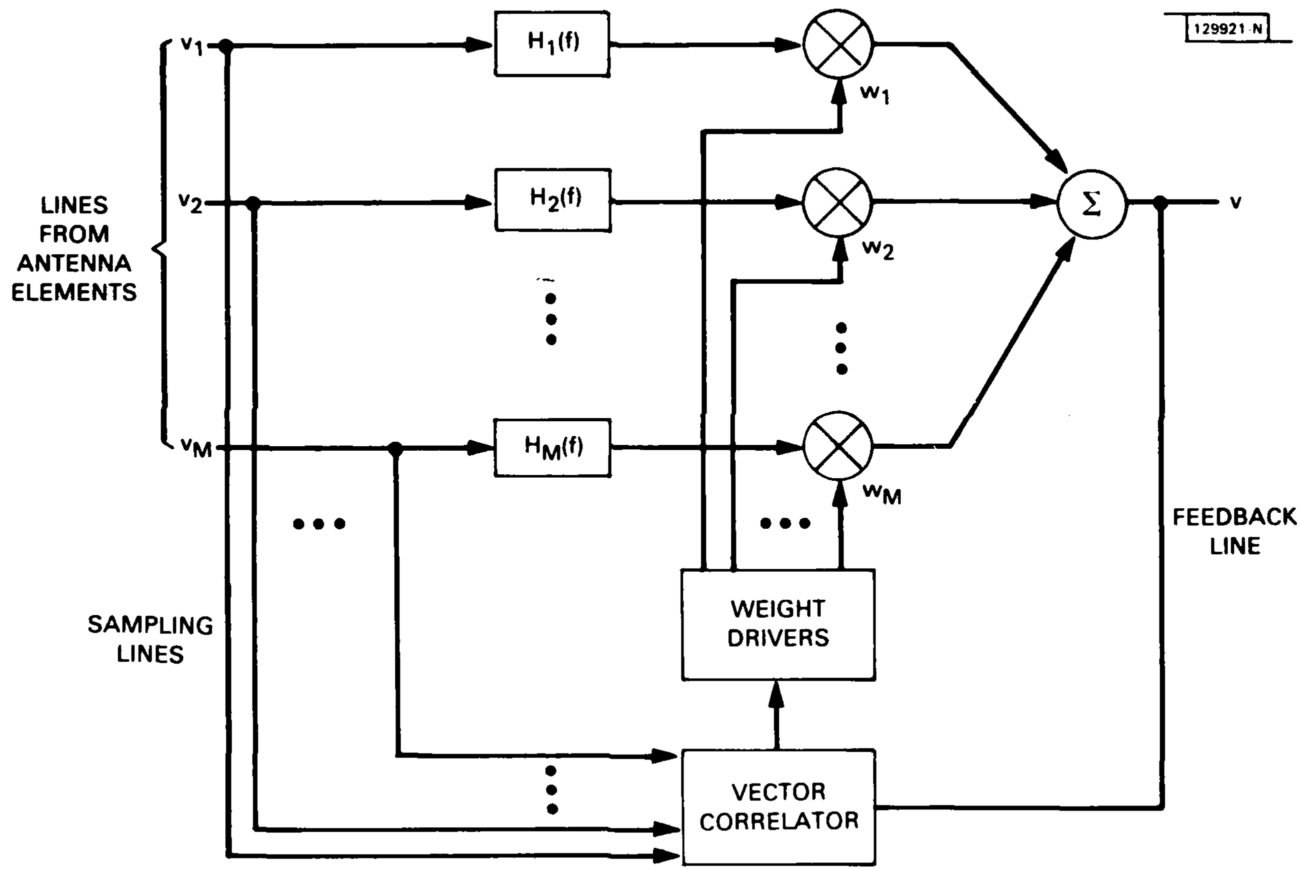

Fig. 1. Howells-Applebaum adaptive circuit showing the channel transfer functions.

wholly compensated for by the adaptive algorithm. In this case, Eq. (1) is to be interpreted as an upper bound. The exact value of the cancellation will depend upon the number of interference sources and the complexity of the mismatch.

In the cases of the direct matrix inversion algorithm[5] and the differential steepest descent (weight-dither) algorithm,[2] these remarks apply to every contributor to the channel mismatch. That is because, in these cases, no part of each signal channel is omitted in deriving successive performance estimates. The algorithm is able, therefore, to compensate to some degree for the channel mismatch.

It is clear that Eq. (1) does not apply to another class of antennas important in adaptive nulling, namely, the multiple-beam antenna (MBA). For example, it is possible in principle to construct a seven-beam MBA in which six beams are arranged uniformly in a circle around the central beam, and in which each of the peripheral beams has a gain of zero in the direction of the beam peak of the central beam. An interference source lying in this same direction can be perfectly cancelled by simply turning off the central beam. Channel tracking errors have no effect in this case.

In the next section, the formula for an MBA corresponding to the array formula, Eq. (1), is presented (it is derived in the Appendix). As expected, it turns out to be more complicated. Numerical simulations show that it is accurate, however, subject to the same conditions of interpretation discussed above for the array formula. One additional restriction is that, analytically, it is valid for only one interference source. To what extent it can be useful for more than one has yet to be investigated.

As might be expected, the granular nature of the gain coverage provided by an MBA is reflected in the depth formula. That is, the attainable cancellation depends on the source location. Other significant variables are the sidelobe structure of the individual beams and the number of beams. The effect of these variables is discussed in Section III.

II. MBA CANCELLATION FORMULA

If the MBA has M beams and the interference voltage v_m appearing at beam port m is given by $W^{1/2}g_m$, where W is the interference power flux density incident upon the adaptive array and g_m is the voltage gain of beam m, then the cancellation of the jammer power is given by

$$C = \frac{S_2^{\,3} - 2\,\mathrm{Re}\{S_1 S_2 S_3^{\,*}\} + |S_1|^2 S_4}{|S_1|^2 S_2 (S_2 - |S_1|^2/M)}\,\sigma_t^{\,2} \tag{2}$$

where the S_k are the sums

$$S_k = \begin{cases} \sum\limits_m g_m & , \quad k = 1 \\ \sum\limits_m |g_m|^k & , \quad k = 2 \text{ and } 4 \\ \sum\limits_m |g_m|^2 g_m, & \quad k = 3, \end{cases} \tag{3}$$

(the summation index runs from 1 to M), and σ_t is the rms variation of the channel tracking errors. This relationship is derived in the Appendix under the assumptions that the tracking errors in any one channel are uncorrelated with those in any other channel, that their rms variation is the same in all channels, that there is a single interference source, and that the quiescent weight vector (or steering vector) has all its components equal. This last assumption is equivalent to assuming that the adaptive antenna is operating in the area-coverage mode, since in the absence of an interfering source, all beams would then be equally weighted.

Since both numerator and denominator of (2) are of degree six in the g_m, the cancellation is independent of the normalization adopted in their definition. It depends only on their relative values.

In the ideal case, the g_m of an MBA could all be purely real. Then (2) reduces to

$$C = \frac{S_2^{\,3} - 2S_1S_2S_3 + S_1^{\,2}S_4}{S_1^{\,2}S_2\left(S_2 - S_1^{\,2}/M\right)}\,\sigma_t^{\,2} \tag{4}$$

where $S_k = \sum_m g_m^{\,k}$.

The results of a numerical check on the accuracy of this cancellation formula is shown in Fig. 2. A nine-beam MBA was simulated in which the channel transfer function mismatch was *statistically identical but independent* in each channel. The beams, of shape $2J_1(x)/x$, were arranged in a three-by-three square array with the four corner beams pointing in all combinations of the azimuth/elevation coordinates (±1.2°, ±1.2°). The channel mismatch was simulated by defining the channel transfer function $H_m(f)$ as a finite Fourier series over the bandwidth of interest and then selecting the complex coefficients h_{mg} using a zero-mean gaussian random-number generator. That is,

$$H_m(f) = 1 + \sum_{q=-Q}^{Q} h_{mq} \exp\left[i2q\pi\left(f - f_o\right)/B\right] , \tag{5}$$

where f_o is the center frequency and B the bandwidth. The rms variation σ_t of the transfer function is given, therefore, by

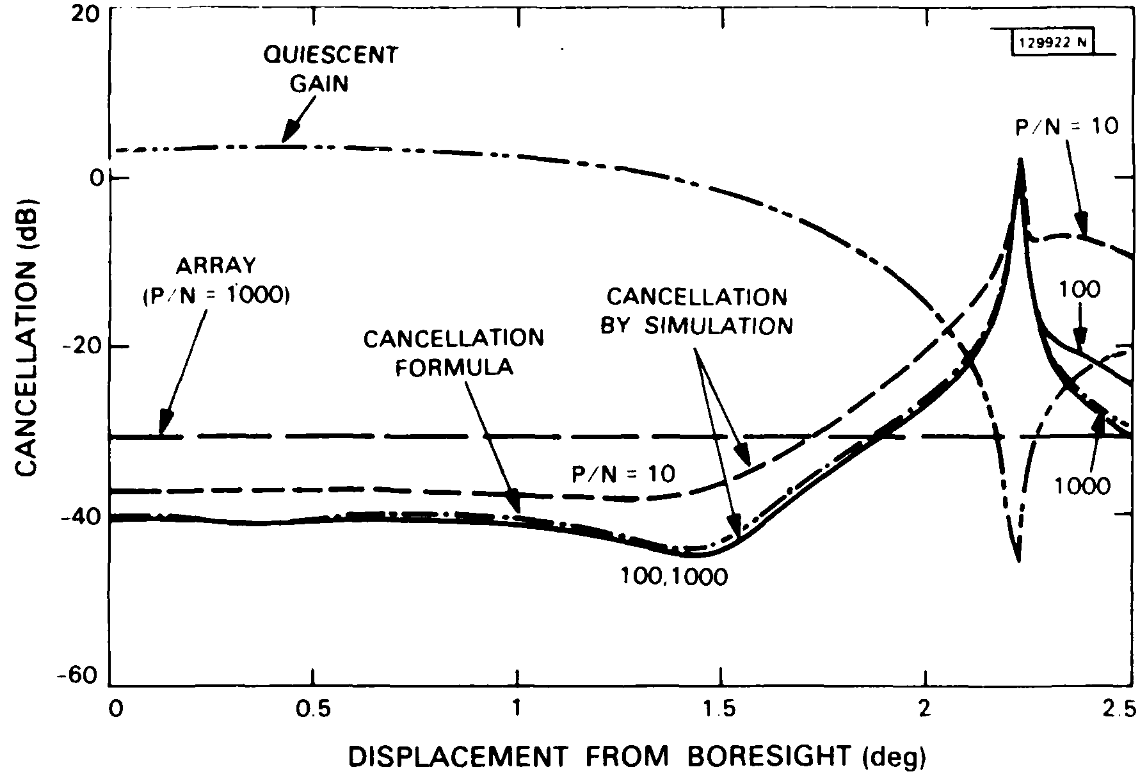

Fig. 2. Cancellation versus source location for a 9-beam MBA. Solid curves are the results of simulations, dot-dash curve is from cancellation formula. Dotted curve is normalized quiescent gain pattern. Dashed curve shows cancellation for a nine-element array having the same tracking errors.

$$\sigma_t^2 = \langle |H_m(f) - 1|^2 \rangle$$

$$= \sum_{q=-Q}^{Q} \langle |h_{mg}|^2 \rangle \quad ,$$

where the angle brackets $\langle \cdot \rangle$ denote expected value. This result depends on the assumption that $h_{mo} = 0$ for $1 \leq m \leq M$, which implies that the average gains of the channels over the operating bandwidth have been equalized.

The real and imaginary parts of each coefficient h_{mg}, for $|q| > 0$, are independent samples having zero mean and variance σ_h^2. Thus

$$\sigma_t^2 = 4\, Q\, \sigma_h^2 \quad . \tag{6}$$

That all the samples are independent also ensures that the simulated tracking errors are statistically uncorrelated from channel to channel.

If σ_t is small compared with unity, then the rms variation σ_{dB} in dB of the channel transfer-function amplitude is given by

$$\sigma_{dB} \simeq 6.14\, \sigma_t \quad , \tag{7}$$

and the corresponding phase variation σ_{deg} in degrees is given by

$$\sigma_{deg} \simeq 40.51\, \sigma_t \quad . \tag{8}$$

Thus, for example, if an adaptive array antenna had uncompensated channel tracking errors of 0.2 dB and 1.32° rms, then from (7) and (8), these numbers imply a value of σ_t of 3.26×10^{-2}. The achievable cancellation is, therefore, from (1), 1.06×10^{-3} or -29.7 dB.

In Fig. 2, there are five cancellation curves plotted. Three are the results of MBA simulations with the same set of channel mismatch coefficients, the three differing only in the power level of the interference source. (P/N is the ratio of the interference to system noise power ratio at the antenna output when the antenna is in its quiescent state.) There were 40 terms in the series for the transfer function of each channel, having a total variance of 3.26×10^{-2}, implying rms channel tracking errors of 0.2 dB in amplitude and 1.32° in phase. The fourth cancellation curve is plotted from the MBA cancellation formula, Eq. (4), and the fifth is the cancellation obtained by simulating an array antenna having nine omnidirectional elements and with tracking errors identical to those of the MBA. The sixth curve, not a cancellation curve, is the quiescent gain pattern of the MBA with arbitrary normalization.

All the curves show the variation of cancellation or gain as a function of the displacement of the source from boresight along a main diagonal of the square beam pattern. The abscissa is calibrated in degrees azimuth or elevation. (For movement along the diagonal, the two are equivalent.)

The four notable features of Fig. 2 are

- the agreement between the cancellation calculated from the MBA cancellation formula and the cancellation measured by the simulation steadily improves as the source power increases, showing, as expected, that the formula is a large-power approximation.
- when the source power is large, the agreement is excellent, showing that the formula is accurate.

- the cancellation of the MBA shows significant dependence on source location.
- the cancellation for the MBA is about 10 dB better, in this case, than that for the array, when both antennas have the same tracking errors.

The gain patterns along the main diagonal of the individual beams are shown in Fig. 3. These show, characteristically for an MBA, that at any source position there are a small number of dominant beams with the remainder involved at their sidelobe levels. Of particular interest, however, is the situation existing near an azimuth (or elevation) of 1.5°. There, only one beam (beam 5), is dominant, and the sidelobe levels of five of the other beams (beams 2, 4, 6, 8, and 9) are more than 30 dB below the level of the dominant beam. It is at this location that, in Fig. 2, the cancellation shows a shallow minimum. This is consistent with the intuitive assessment that tracking errors will be less significant when the adaptive suppression can operate largely by simply turning off one beam.

As a check on this explanation, a second set of simulations was run using again a nine-beam MBA, but this time, the individual beams had unrealistically low sidelobes. Specifically, each beam had the exponential shape $\exp\{-(x/2)^4\}$. The same channel transfer functions were used as for Fig. 2. The resulting cancellation curves are shown in Fig. 4, and the gain patterns of the individual beams in Fig. 5.

A comparison of Figs. 2 and 4 shows the sidelobe structure of the beams to have a potentially major influence on the achievable cancellation. However, this influence exists strongly for some source locations and is

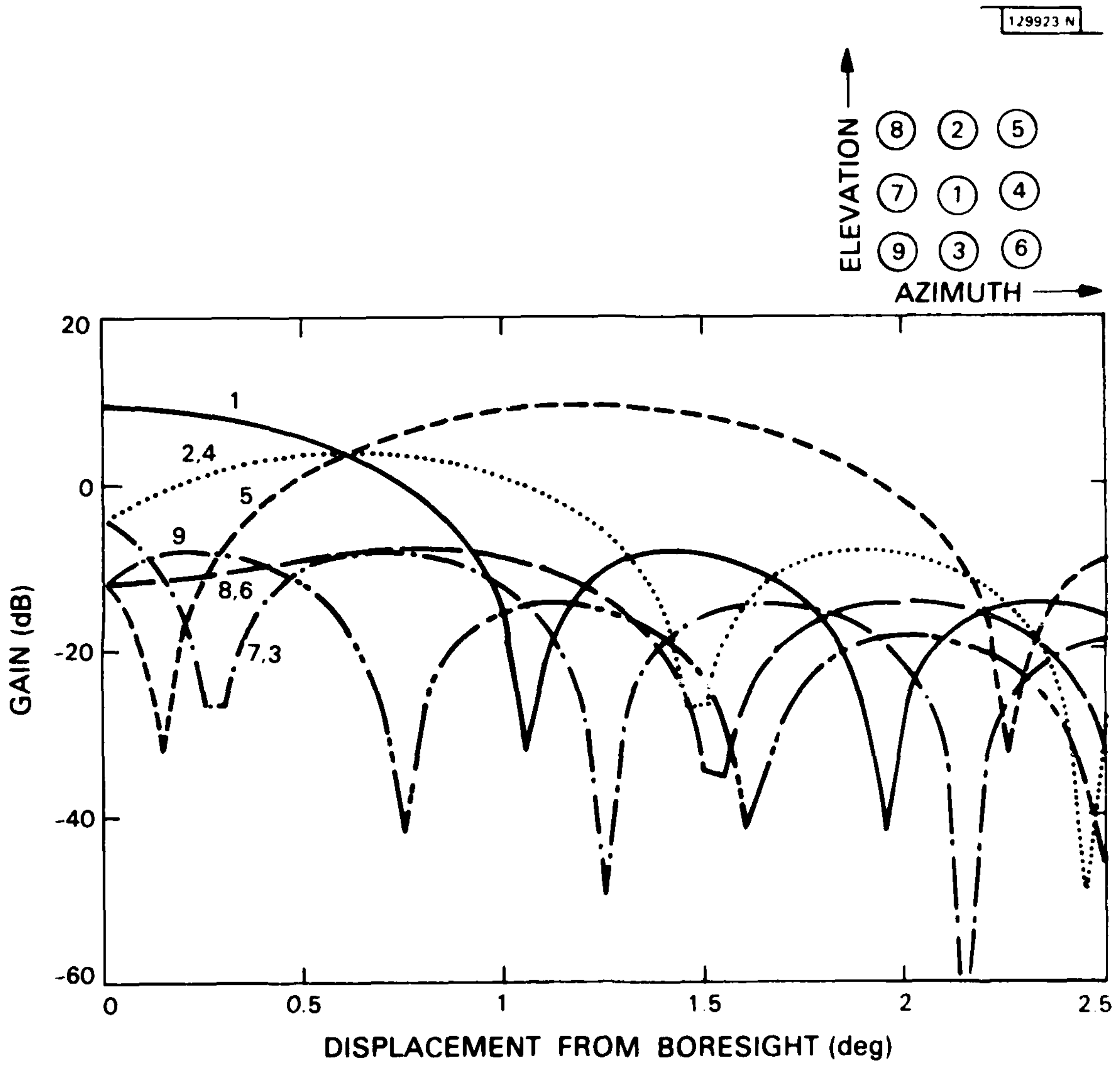

Fig. 3. Gain variation of individual beams along the main diagonal of the square beam pattern.

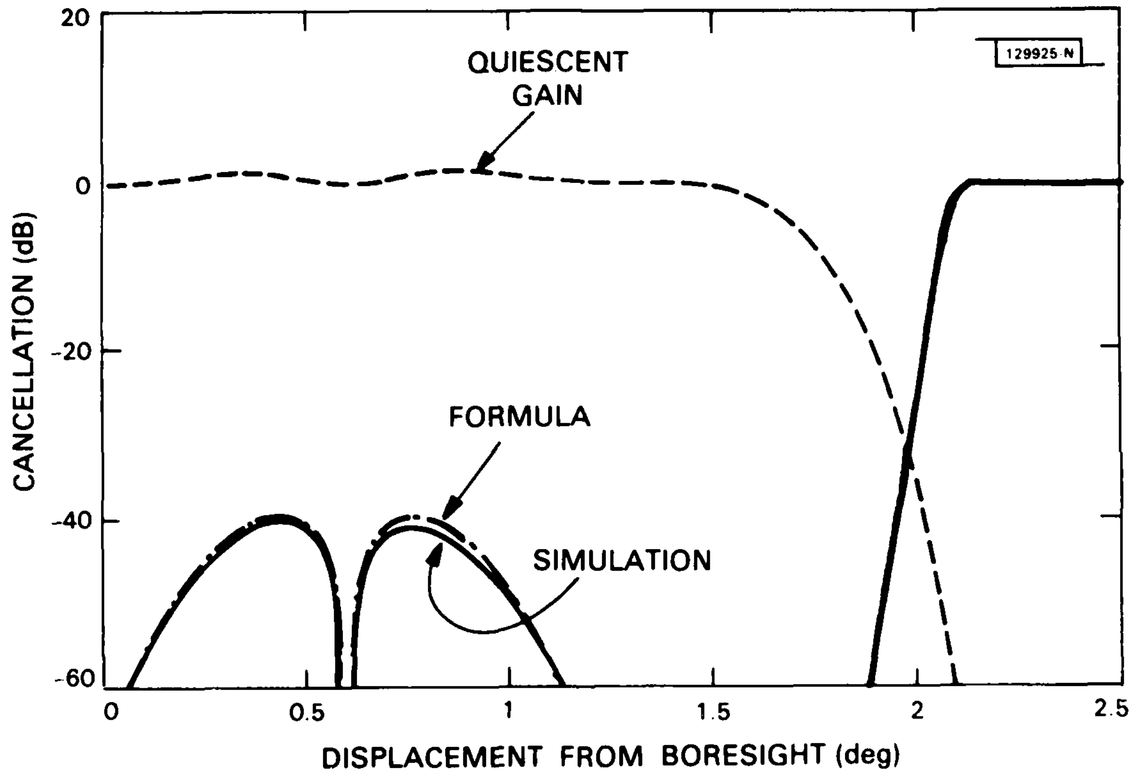

Fig. 4. Cancellation versus source location for a 9-beam MBA using beams with very low sidelobes. Solid curve is the result of simulation, dot-dash curve is from cancellation formula. Dotted curve is quiescent gain pattern.

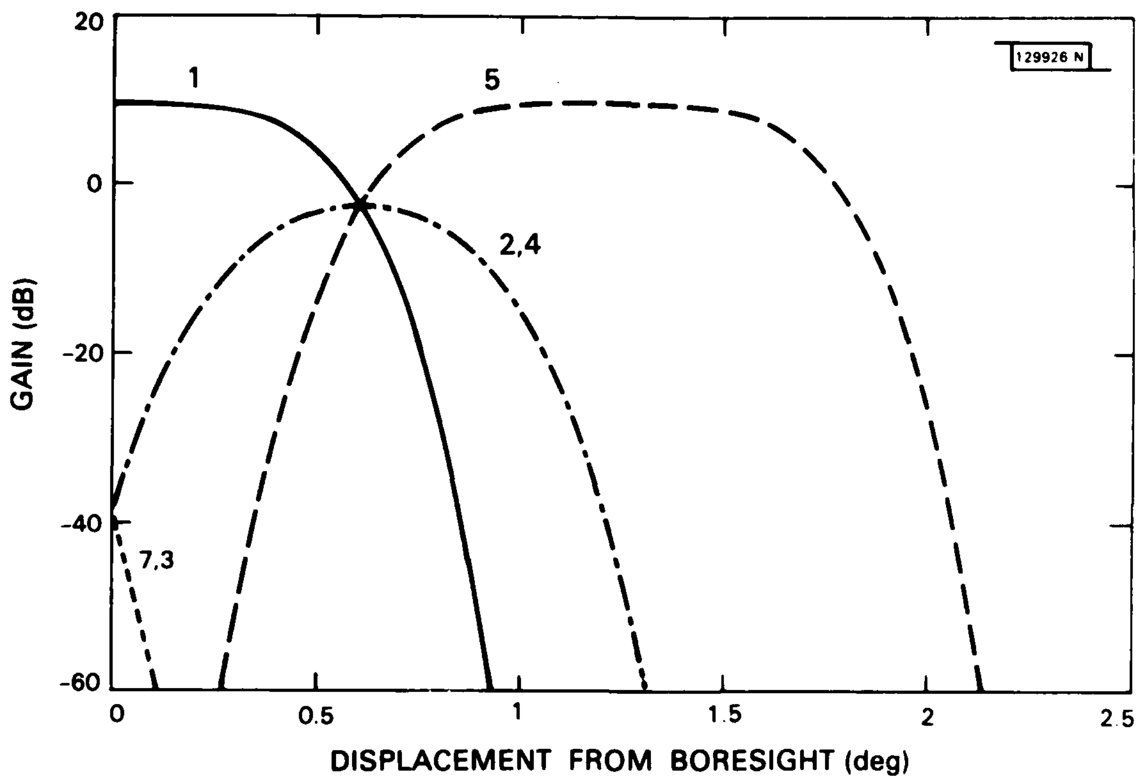

Fig. 5. Gain variation of individual beams along the main diagonal of the square beam pattern formed from beams having very low sidelobes.

essentially absent at others. Such locations are at azimuths of 0 and 0.45°, respectively, for example. This behavior is discussed in the next section.

The two cancellation curves in Fig. 4 demonstrate again the accuracy of the MBA cancellation formula. However, in this case, as for the one illustrated in Fig. 2, the channel tracking errors are described by a 40-term Fourier series for each channel. In the simulations, therefore, there is present a higher degree of statistical smoothing than would obtain were the series to have fewer terms.

To assess the role played in cancellation by the complexity of the mismatch, the simulation was repeated using a four-term series for the mismatch in each channel. The result is shown in Fig. 6. The antenna is the original 9-beam MBA used for the simulation results plotted in Fig. 2 and having the beam shapes of Fig. 3. The variance of the coefficients in the mismatched series was increased by the factor 40/4 to keep the same total rms variation of the transfer function.

There is no essential change in these four-term results from the 40-term results shown in Fig. 2. The discrepancy between the cancellation formula and the actual simulations is larger now, but the formula remains a useful guide to performance.

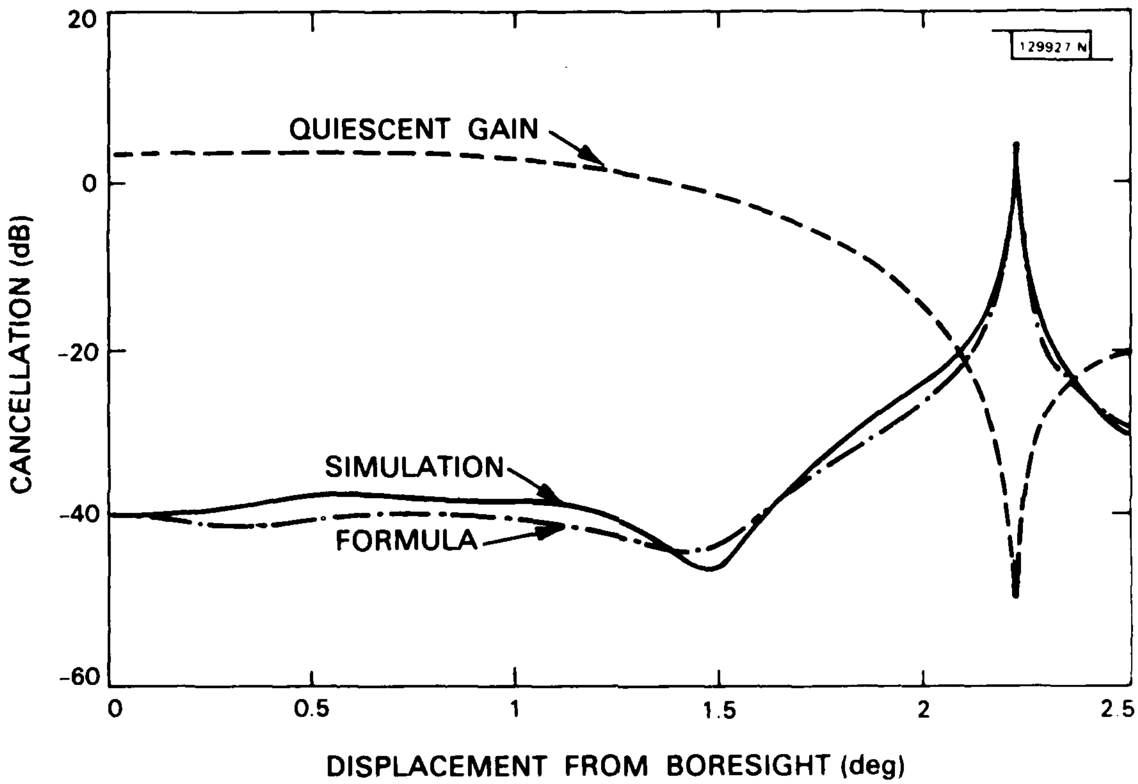

Fig. 6. Cancellation versus source location with only five terms in channel-mismatch series.

III. DISCUSSION

The MBA cancellation formula shows, as might be expected, that when the interference source lies at or near the peak of one beam, the achievable cancellation is very good provided the sidelobe levels of the other beams, in the source direction, are low. The very pronounced null at 0° displacement in the cancellation curves in Fig. 4 illustrates this. There is also another null in these curves located at the point of symmetry between four beams. (The coordinates are 0.6° azimuth and 0.6° elevation.) The existence of this null is less to be expected. However, a moment's reflection makes it clear that this between-beam null arises in the same way as the beam-maximum null. The only difference is that for the between-beam null, the algorithm must turn off more than one beam.

It may be recalled that, in forming a null on an interference source, the algorithm subtracts, from the quiescent antenna pattern, the maximum-gain beam the antenna is capable of pointing at the source. By proper weighting of this cancelling beam, the interference is suppressed. Thus, if the source lies at the peak of a single elemental beam, that beam itself constitutes essentially the maximum-gain cancelling beam. At the point of symmetry between beams, the maximum-gain beam is formed by weighting equally the dominant beams having equal gain the direction of the source. Thus for the beam-maximum source location, one beam is turned off, and for the symmetrical between-beam location, the complete set of two, three, or four beams is turned off.

These considerations lead to the conclusion that, provided the sidelobes of the elemental beams are low, the achievable cancellation is least good when

the source is located between these points of symmetry. In such a location, the cancellation is determined by a small number of dominant beams having unequal gains in the direction of the source. This suggests that Eqs. (2) and (4) for the cancellation of an MBA can be simplified. The simpler expression would apply to an MBA having elemental beams with low sidelobes, and it would give the cancellation to be expected at the source location of poorest cancellation. Thus we are led to examine the cancellation formula subject to the simplifications that there are only m non-zero values of g_m, corresponding to the m dominant beams, and that M >> m.

The result is that, under these conditions, the cancellation is least good when one of the m non-zero g_m is larger than the others, and all the others are equal. Table I presents the magnitude of the smaller beam gains, relative to the largest, which give the poorest cancellation, together with the evaluated cancellation, for different values of m.

TABLE I

WORST-CASE LESSER-BEAM GAINS AND THE RESULTING CANCELLATION VERSUS NUMBER OF DOMINANT BEAMS

No. of dominant beams	Relative gain of lesser beams (dB)	C/σ_t^2 (dB)
2	-9.4	-13.5
3	-11.1	-11.3
4	-12.8	-10.1

The numbers in the table show that an MBA with low-sidelobe elemental beams can achieve, at worst, a cancellation 11.3 or 10.1 dB better than an

array, depending on whether the beams are arranged in a hexagonal or square configuration.

The results plotted in Figs. 4 and 5 are consistent with this conclusion. For the displacement at which the cancellation is least good (about 0.44°), the cancellation is about 10 dB better than that (-29.7 dB) of an array, and the two less dominant beams are smaller than the dominant one by about 11 dB.

The simple result stated above, that the MBA can achieve, at worst, a cancellation some 10 dB better than an array, depends upon the elemental beams having low sidelobes. What "low" means, quantitatively, requires more examination. However, the cancellation results obtained in simulating a nine-beam MBA with $2J_1(x)/x$-shaped beams, shown in Fig. 2, suggest that the low-sidelobe requirement is not too stringent in practice. These results show the 10 dB improvement of the MBA with respect to the array, and yet the gain pattern of each elemental beam does not exhibit especially low sidelobes.

Another, more realistic, example is shown in Fig. 7. This is the two-dimensional contour plot of the cancellation of a 16-beam offset Cassegrain MBA, assuming the rms tracking errors are 0.2 dB in amplitude and 1.32° in phase. The contours were determined by the MBA cancellation formula, Eq. (2), in which the complex gain parameters g_m were evaluated using physical optics to model mathematically the reflection from the two reflectors of the antenna. The 16 beams lay in a 4-by-4 square pattern approximately 2° square.

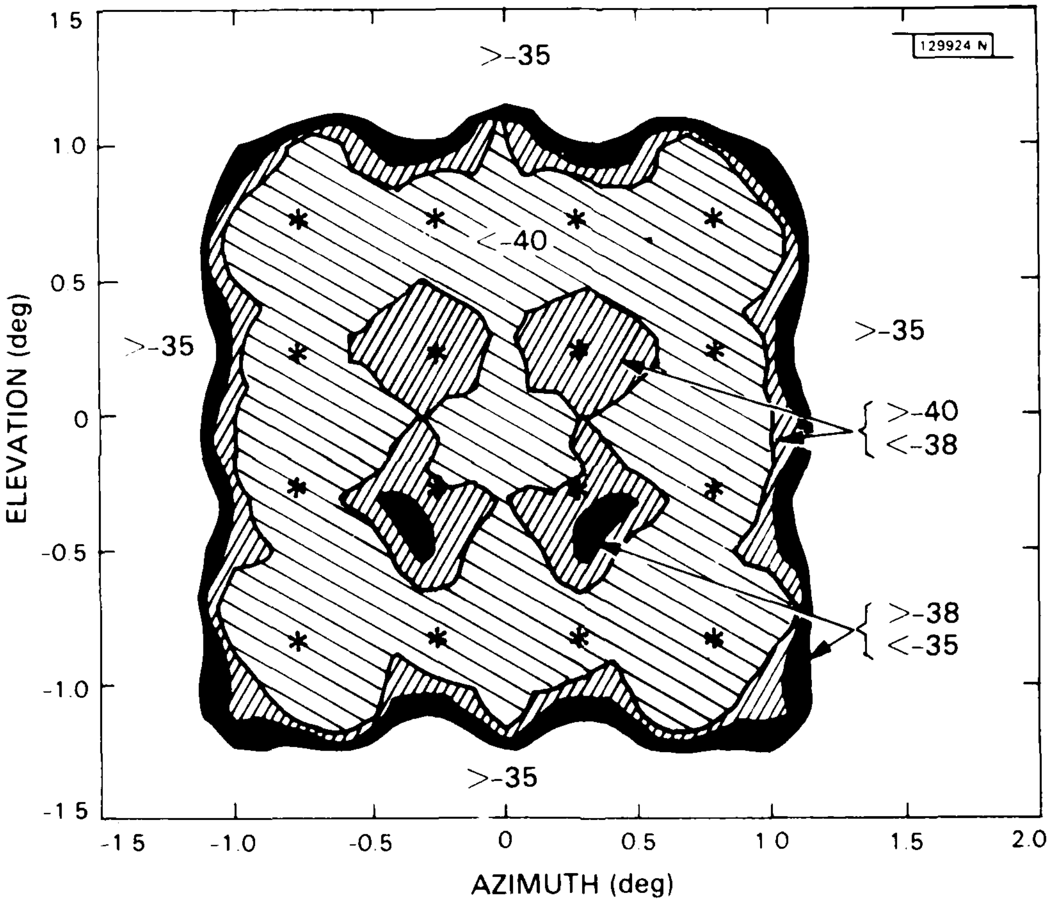

Fig. 7. Cancellation contours in dB for a 16-beam offset Cassegrain MBA. The beam centers are indicated by asterisks. The rms tracking errors are assumed to be 0.2 dB and 1.3°.

The cancellation contour plot shows that within the 2°-square footprint of the antenna, the cancellation is, almost everywhere, better than -38 dB. This is close to the figure of -39.7 dB given by the rule "10 dB better than an array". So here, too, the sidelobes are essentially low enough for the rule to be valid.

The simple "10 dB better" rule was derived by assuming that the total number of beams is large compared to the number of dominant beams in the source direction. If this assumption is not true, then the simple rule may not be true. The difficulty arises at the between-beam point of symmetry, where the cancellation can be very good if the sidelobes of the other beams are low. That is because the interfering source can be rejected by simply turning off the dominant beams. But in the circumstance, for example, that there are only three beams in total, the algorithm would not turn off those three beams to reject an interfering source placed at the central point of symmetry between them. For then, the antenna would reject every source, including the desired ones. Rather, the algorithm sets the weights to give a non-zero sum of their squared magnitudes, but zero for the algebraic sum of the weights. Then the interference source is cancelled, but the antenna maintains some sensitivity in directions away from the source. (The precise values of the weights are determined by the differences, however small, between the transfer functions of the separate channels.)

When the number of beams is small, therefore, the adaptive MBA must depend upon cancellation between the beams as its principal method for rejecting an inteference source. Because of this, channel tracking errors constrain more markedly the null depth achievable by MBA's with a small number

of beams. In fact, the between-beam point of symmetry for the location of an interference source represents a worst case for the cancellation for this type of MBA. This is in contrast to the MBA with many beams, for which the same location is potentially one of the best cases, as Fig. 4 shows.

A demonstration of this behavior is presented in Fig. 8, which shows the cancellation as a function of source location for a four-beam MBA. The antenna is the nine-beam MBA used to produce the results in Fig. 4, but five of the beams have been deleted. Only the beams at the azimuth/elevation coordinates (0,0), (0, 1.2), (1.2, 0), and (1.2, 1.2) remain. (The units are degrees.) These are the beams numbered 1, 2, 4, and 5 in the sketch in Fig. 3. The individual beam shapes, along the line of equal azimuth and elevation, are shown in Fig. 5. The source was moved along this same line to obtain the cancellation curves of Figs. 4 and 8. A comparison of these two sets of cancellation curves at 0.6°, the point of symmetry, clearly shows the conversion from a best case to worst case when the number of beams is reduced from 9 to 4.

This profound difference in cancellation would not be apparent in practice, because real MBA beams do not have the low sidelobes of the special beam shapes used to generate the data in Figs. 4 and 8. Figure 2 shows that, with a more realistic beam shape, there is no deep cancellation minimum at 0.6°. The special beams were chosen to emphasize the particular phenomenon being demonstrated.

The peak of the cancellation curves in Fig. 8 for the four-beam MBA is greater than that of the curves in Fig. 4 for the nine-beam MBA. Thus the

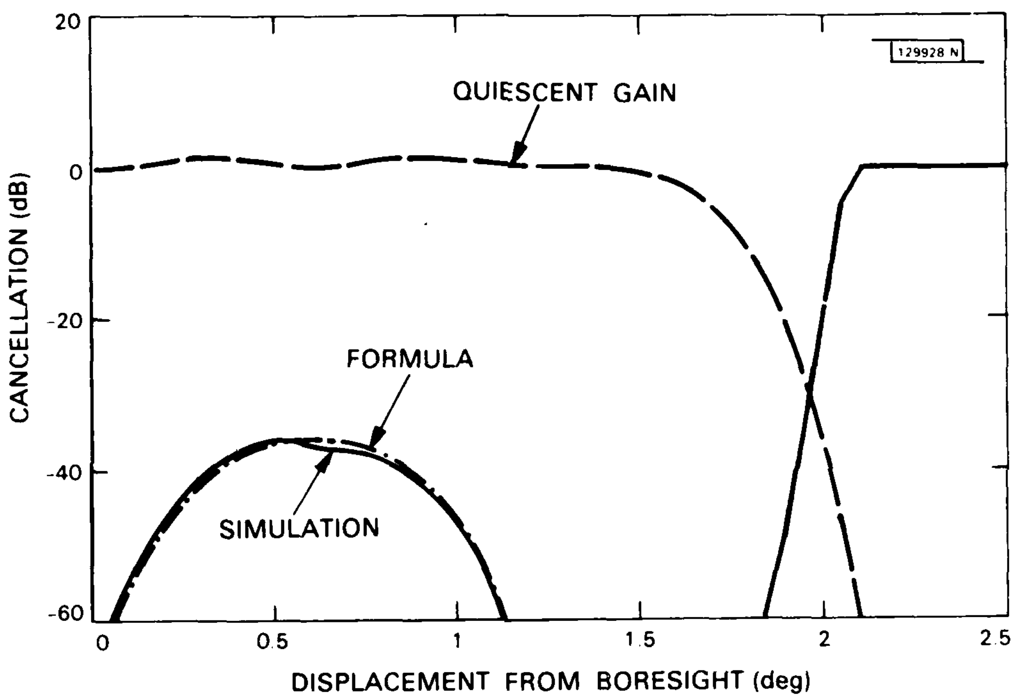

Fig. 8. Cancellation versus source location of the 4-beam MBA generated by deleting all but beams 1, 2, 4, and 5 of the 9-beam MBA used for Fig. 4. Solid curve is the result of simulations, the dot-dash curve is from the cancellation formula. The dotted curve is the quiescent gain pattern.

simple rule "10 dB better than an array" no longer applies when the number of beams of the MBA is small enough for there to be a between-beam point of symmetry involving all beams of the MBA at equal amplitude.

An analysis of this situation in the Appendix concludes that at this point of symmetry, the cancellation is given~~ by~~

$$C = \frac{\sigma_t^2}{M} \quad . \tag{9}$$

As before, σ_t is the rms variation of the uncompensated channel tracking errors and M is the total number of beams. The curves in Fig. 8 agree with this formula, since their peak is about -36 dB, which is about 6 dB less than the -29.7 dB value for an array. (Constant in all these comparisons is the assumption that σ_t is 3.26×10^{-2}, implying rms channel tracking errors of 0.2 dB and 1.32°.) Since M = 4 in this case, Eq. (9) accounts properly for the 6 dB difference.

IV. CONCLUSIONS

Analysis and computer simulations confirm the intuitive judgment that an adaptive MBA can achieve greater rejection of a wide-band interference source than an array when both have the same channel tracking errors. The reason for its better performance is that the MBA can effect substantial rejection of the source by simply turning off the beam or beams principally affected by the source. Thus the need to cancel the interference signal in one channel with that in another is felt only at the sidelobe level of the remaining beams. The array, on the other hand, depends totally on cancellation to provide its rejection. The channel tracking errors, which prevent perfect cancellation, therefore, allow less interference power to leak through the MBA.

In the case of a single interference source together with antenna operation in the area-coverage mode, the cancellation of the source (that is, the ratio of the interference power at the antenna output after adaption to its value before adaption) is given, in the limit of large source power, by σ_t^2 for an array and has an upper bound of about $\sigma_t^2/10$ for an MBA. Here σ_t is the rms channel tracking error of each channel. The cancellation is independent of source location for an array, but not for an MBA.

The MBA formula depends on the sidelobes of the elemental beams being "low" and the total number of beams being "not small", in the sense defined in the previous sections. Neither of these restrictions should present any difficulties in practice. The sidelobe levels of the elemental beams of an MBA are expected normally to be low enough, in this sense, without any special design care. And for conventional beam arrangements, the "not small"

restriction boils down to there being more than four beams, for a square arrangement, or three beams, for a triangular arrangement.

If the number of beams is "small", in the sense used here, the cancellation has an upper bound of σ_t^2/M, where M is the number of beams.

The difference between the "small" and "not small" cases could be of importance in interpreting null depth measurements made using only two, three, or four of the total number of beams. This situation could occur during early testing in the design evaluation of an antenna ultimately intended to have many beams. If only two beams were used as part of a test of principle, for example, the resulting measurements would show the cancellation to be $\sigma_t^2/2$, if the interference source were at the between-beam point of symmetry. However, the achievable cancellation of a many-beam system using these components would be $\sigma_t^2/10$. A correction factor of 7 dB would have to be applied, therefore, to the two-beam results to predict the performance of the many-beam system.

V. ACKNOWLEDGMENTS

Thanks are due to my colleagues Andre Dion, for his mathematical model of a 16-beam offset Cassegrain MBA; Lee Niro, for combining the model and Eq. (2) to produce the resulting cancellation contours in Fig. 7; and Alan Simmons and William Cummings for their stimulation and support of this work.

VI. APPENDIX

In this Appendix, the MBA cancellation formula is derived. The assumptions are that there is one interference source, and that the channel tracking errors can be lumped in the transfer functions $H_n(f)$ shown in Fig. 1. Thus aperture dispersion, and tracking errors in the feed lines external to the adaptive loops, are assumed to be negligible.

Figure 1 would also seem to imply that tracking errors in the sampling lines and feedback line are also assumed to be negligible. The implication is valid, but for these tracking errors, no approximation is involved in the assumption. This is because the attainable null depth depends only on tracking errors in the signal lines. The feedback and sampling lines can include arbitrary linear filters without affecting the achievable null depth. Thus any tracking "errors" in these lines, however large, are negligible.

The Howells-Applebaum adaptive algorithm, when fully adapted, generates the adapted weights w_n as the solutions of the equation

$$\langle v_m^* v \rangle + N\, w_m = N\, w_m^{(0)} \quad . \tag{10}$$

Here, N is the rms system noise voltage, the $w_n^{(0)}$ are the components of the steering vector, the v_m are the interference voltages at the output ports of the antenna elements or feeds, and v is the output voltage from the summing port of the antenna. Thus

$$v = \sum_{m=1}^{M} w_m H_m v_m \quad . \tag{11}$$

The angle brackets denote time average.

If W is the interference power flux density incident on the antenna and the set g_n, for n = 1, 2, ..., M, are the voltage gains of the individual elements or beams, then (10) can be rewritten as

$$W g_n^* \sum_m g_m \langle H_m \rangle w_m + N\, w_n = N\, w_n^{(0)} \quad , \tag{12}$$

where the summation index runs from 1 to M, M being the number of channels. The interference power is assumed to be spectrally flat over the operating bandwidth of the antenna. This assumption, together with the assumption of negligible aperture dispersion, allows the time average in (10) to be replaced by the frequency average $\langle H_m \rangle$ in (12). Further, since for simplicity we assume that the average amplitude and phase responses of the channels have already been matched, then adopting (5) as the definition of H_m, we find $\langle H_m \rangle = 1$. Thus (12) becomes

$$\frac{W}{N} g_n^* \sum_m g_m\, w_m + w_n = w_n^{(0)} \quad . \tag{13}$$

To solve this for w_n, first we evaluate $\sum g_m w_m$ by multiplying through by g_n, summing over n and rearranging. Substituting the resulting expression for $\sum g_m w_m$ back in (13), we find

$$w_n = w_n^{(0)} - \beta g_n^* \quad , \tag{14}$$

where

$$\beta = \frac{\sum_m w_m^{(0)} g_m}{\frac{N}{W} + \sum_m \left| g_m \right|^2} \quad . \tag{15}$$

The interference power output P from the antenna is given by $\langle |v|^2 \rangle$. Thus, from (11),

$$\frac{P}{W} = \frac{\sum_{n,m} w_n^* w_m g_n^* g_m \langle H_n^* H_m \rangle}{\sum_m |w_m|^2} \tag{16}$$

for a unity-magnitude weight vector. (Normalizing the weight vector in this way makes valid the use of a constant value of N to denote the system noise level.) From (5), the frequency average $\langle H_n^* H_m \rangle$ can be expressed as

$$\langle H_n^* H_m \rangle = 1 + \sum_q h_{nq}^* h_{mq} \ .$$

But since the tracking errors are uncorrelated from channel to channel, the *summation will be close to zero unless n = m.* Thus we make the approximation

$$\langle H_n^* H_m \rangle = 1 + \delta_{nm} \sigma_t^2 \ , \tag{17}$$

where δ_{nm} is the Kronecker delta and σ_t is the rms variation of the tracking errors.

Before adaption, the power output P_o from the antenna is given by (16) but with the w_n replaced by $w_n^{(0)}$. Thus

$$\frac{P_o}{W} = \frac{\left|\sum_m w_m^{(0)} g_m\right|^2}{\sum_m \left|w_m^{(0)}\right|^2} \ . \tag{18}$$

The fact that the tracking errors have negligible effect on the power output <u>before</u> adaption simplifies (18) by allowing the substitution $H_n = 1$.

Finally, the cancellation C, defined as P/P_o, is given from (16), (17), and (18) by

$$C = \frac{\sum_{n,m} w_n^* w_m g_n^* g_m \left(1 + \delta_{nm} \sigma_t^2\right)}{\left|\sum_m w_m^{(0)} g_m\right|^2} \cdot \frac{\sum_m \left|w_m^{(0)}\right|^2}{\sum_m \left|w_m\right|^2} \tag{19}$$

where the w_n are given by the limiting case as W/N approaches infinity of (14) and (15). But in this limiting case, the w_n satisfy

$$\sum_m w_m g_m = 0 \quad ,$$

as can be verified by multiplying (14) by g_n and summing over n, and so (19) simplifies to

$$C = \frac{\sum_m \left|w_m g_m\right|^2 \sum_m \left|w_m^{(0)}\right|^2}{\left|\sum_m w_m^{(0)} g_m\right|^2 \sum_m \left|w_m\right|^2} \; \sigma_t^2 \; , \tag{20}$$

with

$$w_n = w_n^{(0)} - \frac{\sum_m w_m^{(0)} g_m}{\sum_m \left|g_m\right|^2} \; g_n^* . \tag{21}$$

For the MBA in the area coverage mode, $w_m^{(0)} = 1$ for all m. Substituting these values in (20) and (21), we obtain, after some manipulation, the MBA cancellation formula (2) in Section III.

Similarly, for the array in the area coverage mode, $w_m(0) = \delta_m$, and $|g_m|$ is independent of m. Substituting these values in (20) and (21) eventually leads to the known result[3]

$$C = \sigma_t^2 \quad . \tag{22}$$

The general cancellation formula, Eq. (20), can be rewritten in an illuminating form. Since $|g_m|^2$ is proportional to the element (or beam) gain G_m and $\sum w_m(0) g_m$ is proportional to the quiescent gain G_o, (20) can be expressed

$$C = \frac{\sum\limits_m |\hat{w}_m|^2 G_m}{G_o} \sigma_t^2 \tag{23}$$

where w_m is the m'th component of the normalized weight vector. This expression shows that each channel contributes, in effect, independently to the residual interference power emerging from the adaptive antenna. The total residual interference power is expressed as the weighted sum of the interference power carried in each channel separately. Moreover, the weights of this weighted sum are precisely the adapted weights of the variable combiner multiplied by the constant σ_t^2. Thus, when the algorithm turns down the dominant beams of an MBA, the interference power is carried through the sidelobe level of the remaining beams. For the array, on the other hand, G_m is independent of m, which converts (23) to $C = (G_m/G_o)\sigma_t^2$, since w_m is normalized. The interference is carried at the level of the element gain, therefore.

Formula (23) is also convenient for evaluating the cancellation at the between-beam point of symmetry for an MBA with only a small number of symmetrically placed beams. At this point, the G_m are all equal to one another, and $G_o = M\ G_m$. Hence $C = {\sigma_t}^2/M$, the formula given as Eq. (9) in Section III.

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The ability of an adaptive antenna to reject wide-band interference sources is limited by the differences between the transfer functions of the different channels. An examination of the quantitative relationship between the rejection and the tracking errors shows that the multiple-beam antenna (MBA) achieves greater rejection than an array antenna. This is because, in forming a null in the direction of an interference source, the MBA achieves much of its total rejection by simply turning off the channel principally affected by the source. The array, on the other hand, must depend upon cancellation between channels to provide all its rejection.

A simple formula exists, and is presented in the test, relating the interference cancellation to the rms mismatch between the channel transfer functions.

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8